Problem 63

The average density of the Sun is on the order 10^3 kg/m^3. (a) Estimate the diameter of the Sun. (b) Given that the Sun subtends at an angle of about half a degree in the sky, estimate its distance from Earth.

Solution

Part (a)

According to Appendix D on page 894,

$$\text{Mass of Sun :} \quad 1.99 \times 10^{30} \text{ kg.}$$

With the given density, the diameter of the Sun can be calculated.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \rightarrow \quad 10^3 \, \frac{\text{kg}}{\text{m}^3} = \frac{1.99 \times 10^{30} \text{ kg}}{\frac{4}{3}\pi R^3}$$

Solve for R, the radius of the Sun.

$$\frac{4}{3}\pi R^3 = \frac{1.99 \times 10^{30}}{10^3} \text{ m}^3$$

$$R^3 = \frac{3}{4\pi}\frac{1.99 \times 10^{30}}{10^3} \text{ m}^3$$

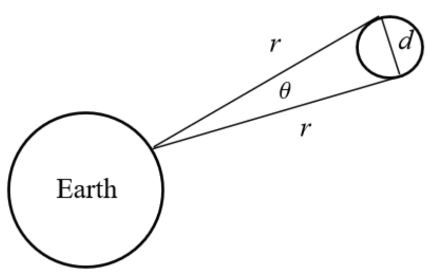
$$R = \sqrt[3]{\frac{3}{4\pi}\frac{1.99 \times 10^{30}}{10^3}} \text{ m} \approx 8 \times 10^8 \text{ m}$$

The diameter of the Sun is double the radius.

$$\text{Diameter of Sun :} \quad 2R \approx 2 \times 10^9 \text{ m}$$

Part (b)

Draw the Earth, the Sun, and the subtended angle θ. Let the distance from the Earth to the Sun be r, and let the diameter of the Sun be d.



The equation relating these variables is the formula for arclength.

$$d = r\theta$$

Solve for r, noting that θ has to be in radians.

$$r = \frac{d}{\theta} = \frac{2 \times 10^9 \text{ m}}{0.5 \times \frac{\pi}{180}} \approx 2 \times 10^{11} \text{ m}$$